HOPF ALGEBRAS AND GALOIS MODULE THEORY, JUNE 4 & 5, 2025

Half-baked ideas are welcome! This has always been a working conference, and over the years, one of the most enjoyable features has been the inclusion of half-baked ideas, guiding principles, and crazy conjectures. Please come with some to share.

Break-out rooms provided. Another valuable feature of this conference has always been the ample time provided for discussion. Zoom break-out rooms will be available after every two talks. One room named for the first talk, one for the second, and one for general conversation.

Zoom link: `https://bostonu.zoom.us/j/99488387760?pwd=tLYkhXSxfuh9C7g6GqLbAghl42MaaI.1`
We intend to record the talks and post them at `www.hopf-galois.org/`. If you would rather not have your talk recorded, please let me know.

Time Zone Conversion. You might find this helpful. I did.

CDT	EDT	**UTC**	BST	CEST	JST
8:00	9:00	**13:00**	14:00	15:00	22:00
12:00	13:00	**17:00**	18:00	19:00	2:00

Wednesday. Moderator: Tim Kohl

13:00UTC Leandro Vendramin, *What is a skew brace?* 50 minutes
14:00UTC Nigel Byott, *An approach to a conjecture of Rump via pointed magmas.* 50 minutes.
BREAK 15:00UTC – Break-out rooms available.
15:30UTC Daniel Gil-Muñoz, *The ring of integers in degree p extensions of p-adic fields.* 50 minutes
16:30UTC Kevin Keating, *Some nonabelian subgroups of the Nottingham group over* $\mathbb{F}_4$. 50 minutes
BREAK 17:30UTC – Break-out rooms available.

Thursday. Moderator: Alan Koch

13:00UTC Cindy Tsang, *Classification of the types for which every Hopf–Galois correspondence is bijective* . 25 minutes
13:30UTC Tim Kohl, *Pauli Groups and Hopf-Galois Structures* . 50 minutes
14:30UTC Andrea Caranti, *A First Sylow Theorem for skew braces?* 50 minutes
BREAK 15:30UTC – Break-out rooms available.
16:00UTC Paul Truman, *On some semidirect products of skew braces arising in Hopf-Galois theory* . 50 minutes.
17:00UTC Rob Underwood, *Extending Harrison's induction map to non-abelian groups.* 50 minutes
BREAK 18:00UTC – Break-out rooms available.

Abstracts

Nigel Byott, University of Exeter
An approach to a conjecture of Rump via pointed magmas
50 minutes
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Coauthors: Edgar Jasko

Abstract: Cycle sets were introduced by W. Rump in 2005 in order to study left nondegenerate unitary solutions of the quantum Yang-Baxter Equation. One way to obtain cycle sets is from an abelian group A and a permutation τ of A satisfying certain conditions. Such cycle sets (A, τ) are called quasi-linear cycle sets. The socle of (A, τ) is a certain subgroup $\mathrm{Soc}(A, \tau)$ such that $A/\mathrm{Soc}(A, \tau)$ is again a quasi-linear cycle set, called the retraction of (A, τ). Rump conjectured that every finite quasi-linear cycle set has non-trivial socle, so that it can be reduced to a set of size 1 by repeatedly taking retractions.

In this talk, I will reinterpret and generalise Rump's conjecture by viewing quasi-linear cycle sets as objects in the category of pointed magmas (i.e. sets with a binary operation and a distinguished element) and considering their automorphism groups in this category. I will then discuss some theoretical and computational evidence in support of this new conjecture.

Andrea Caranti, Università di Trento
A First Sylow Theorem for skew braces?
50 minutes
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Coauthors: Ilaria Del Corso, Maria Ferrara, Marco Trombetti and Massimilano Di Matteo.

Abstract: Does the equivalent of the First Sylow Theorem for groups hold for finite skew braces? Although this may appear unlikely, it might be interesting to consider special cases. We will present a positive answer for the (admittedly very special) case of finite, supersolvable skew braces. Work is ongoing on the more challenging case of finite, solvable skew braces.

Daniel Gil-Muñoz, Charles University & Università di Pisa
The ring of integers in degree p extensions of p-adic fields
50 minutes
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Abstract: Let p be an odd prime number. The problem of characterizing the freeness of the ring of integers in a Galois degree p extension of p-adic fields as a module over its associated order was solved by F. Bertrandias, J. P. Bertrandias and M.J. Ferton. In a recent paper, the author proved an analogous result for degree p extensions of p-adic fields whose normal closure is dihedral of degree $2p$. In this talk, we shall see how to adapt these techniques in order to prove a characterization for an arbitrary degree p extension of p-adic fields with no restriction on the Galois group of the normal closure.

Kevin Keating, University of Florida
Some nonabelian subgroups of the Nottingham group over $\mathbb{F}_4$
50 minutes
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Abstract: Let k be a finite field. The Nottingham group $\mathcal{N}(k)$ over k consists of power series over k of the form $t + a_1t^2 + a_2t^2 + \dots$ with the operation of substitution. Byszewski, Cornelissen, and Tijsma recently showed how to give explicit descriptions of some finite abelian subgroups of Nottingham groups using finite automata. I will use extensions of local fields of characteristic p to give explicit descriptions of some nonabelian subgroups of $\mathcal{N}(\mathbb{F}_4)$.

Timothy Kohl, Boston University
Pauli Groups and Hopf-Galois Structures
50 minutes
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Abstract: The Pauli Groups are a class of groups, denoted P_n, of order 4^{n+1} for $n \geq 1$, which arise from the so-called Pauli spin-matrices, the smallest of which, denoted P_1, has order 16. We consider the Hopf-Galois structures on extensions with Galois group P_1, and type $[P_1]$. We not only consider the enumeration of these structures, but also the implications for braces with additive and multiplicative groups isomorphic to P_1. Of particular interest is the fact that all of these are contained within the holomorph of P_1. We also give some information on what happens with larger P_n.

Paul Truman, University of Keele
On some semidirect products of skew braces arising in Hopf-Galois theory
50 minutes
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Abstract: If L/K is a finite extension of fields which is H-Galois for some K-Hopf algebra H then each Hopf subalgebra H' of H yields an intermediate field L' such that L/L' is $(L' \otimes H')$-Galois. If H' is a *normal* Hopf subalgebra of H then there is a short exact sequence of K-Hopf algebras

$$K \to H' \to H \to \overline{H} \to K$$

such that L'/K is $\overline{H}$-Galois. It is natural to study the corresponding *extension problem for Hopf-Galois structures*: given a finite extension of fields L/K, an intermediate field L', and K-Hopf algebras H' and $\overline{H}$ such that L/L' is $(L' \otimes H')$-Galois and L'/K is $\overline{H}$-Galois, can we construct or classify K-Hopf algebras H that fit into a short exact sequence as above and such that L/K is H-Galois? If we assume that all of the field extensions involved are Galois then we may reinterpret this problem in the language of skew braces: given finite skew braces A and $\overline{A}$, can we construct or classify skew braces G that fit into a short exact sequence of the form

$$1 \to A \to G \to \overline{A} \to 1?$$

We address this question using various notions of semidirect product for skew braces and explore the consequences for Hopf-Galois theory. We identify several existing classification results as instances of our construction.

Cindy (Sin Yi) Tsang, Ochanomizu University, Tokyo
Classification of the types for which every Hopf–Galois correspondence is bijective
25 minutes
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Coauthors: Lorenzo Stefanello
Abstract: For any Hopf–Galois structure H on a finite extension L/K, there is a Hopf–Galois correspondence from the K-Hopf subalgebras of H to the intermediate fields of L/K, which is always injective but not necessarily surjective. We consider the case when L/K is a finite Galois extension. Stefanello and Trappeniers classified the finite groups G such that for any Galois G-extension L/K, the Hopf–Galois correspondence is bijective for every Hopf–Galois structure on L/K. In this talk, I will report on joint work with Lorenzo Stefanello, where using very similar techniques, we classified the finite groups N such that for any Hopf–Galois structure H of type N on any Galois extension L/K, the Hopf–Galois correspondence for H is bijective.

Robert Underwood, Auburn University at Montgomery
Extending Harrison's induction map to non-abelian groups
50 minutes
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Abstract: Let K be a field, let F be a finite abelian group, and let $\mathcal{G}al(K, F)$ denote the set of isomorphism classes of F-Galois extensions of K. Let $U \leq F$ and suppose that $L \in \mathcal{G}al(K, U)$. Let $\mathrm{Map}(F, K)$ denote the trivial F-Galois extension of K. Then $L \otimes_K \mathrm{Map}(F, K) \in \mathcal{G}al(K, U \times F)$ (Auslander and Goldman, 1960). Since F is abelian, there is a homomorphism of groups $\psi : U \times F \to F$, $(u, g) \mapsto ug$, with $(U \times F)/\ker(\psi) \cong F$.

By Chase, Harrison, Rosenberg, 1965, $(L \otimes_K \mathrm{Map}(F, K))^{\ker(\psi)}$ is an F-Galois extension of K. In this way, we define a map

$$T_U : \mathcal{G}al(K, U) \to \mathcal{G}al(K, F),$$

$L \mapsto (L \otimes_K \mathrm{Map}(F, K))^{\ker(\psi)}$, which is the *Harrison induction map* (Harrison, 1965). The main result of Harrison, 1965, states the following: if A is an F-Galois extension of K, then there exists a subgroup $U \leq F$ and a U-Galois extension L/K for which $T_U(L) = A$, i.e., A can be induced from the data U, L.

In the case that F is non-abelian, the construction of the map T_U as above is not possible: the map $U \times F \to F$ may not be a homomorphism. Pareigis, 1988, has addressed this issue and has extended the map T_U to include non-abelian groups. Moreover, the analog of Harrison's main result holds: an arbitrary F-Galois extension A can be induced from some pair U, L, i.e., $T_U(L) = A$ for some $U \leq F$ and some U-Galois extension L/K.

In this talk we review these induction results and relate them to recent work of Kohl and U., 2025.

Leandro Vendramin, Vrije Universiteit Brussel

What is a skew brace?

50 minutes

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Abstract: Skew braces are ring-theoretical algebraic structures to study set-theoretic solutions of the Yang–Baxter equation. A typical example of a skew brace is a Jacobson radical ring. In this talk, we will discuss the basic properties of skew braces and how these structures are related to the celebrated Yang–Baxter equation. We will also discuss connections with groups, categorical algebra and non-commutative rings.