HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 29 – JUNE 2, 2023

Half-baked ideas are welcome! This has always been a working conference, and over the years, one of the most enjoyable features has been the inclusion of half-baked ideas, guiding principles, and crazy conjectures. Please come with some to share.

Breakout rooms provided. Another valuable feature of this conference has always been the ample time provided for discussion. Zoom breakout rooms will be available after every two talks. One room named for the first talk, one for the second, and one for general conversation.

Zoom link: `https://unomaha.zoom.us/j/95655932060?pwd=SGhjTTczb29NVTF2QXowclFaYnVBQT09`
We intend to record the talks and post them at `www.hopf-galois.org/`. If you would rather not have your talk recorded, please let me know.

Time Zone Conversion. You might find this helpful. I did.

CDT	EDT	**UTC**	BST	CEST	JST
8:00	9:00	**13:00**	14:00	15:00	22:00
12:00	13:00	**17:00**	18:00	19:00	2:00

Sunday.

5:00PM CDT Pool Party at 5624 Leavenworth St. Please let me know of any food constraints. I am planning on grilling: brats, etc. There will be salads, etc. And cake/ice cream. This doubles as a B-day party to me and a friend, Lori Elliott-Bartle. Expect to see some of our other friends (nonmathematicians) there – though there are mathematical links: John Bartle is the son of Robert Bartle (real analysis) `https://en.wikipedia.org/wiki/Robert_G._Bartle`.

Monday. Moderator: Andrea Caranti

13:00UTC Tim Kohl, *Beyond Isomorphic Holomorphs.* 50 minutes
14:00UTC Lorenzo Stefanello, *Bi-skew braces in Hopf–Galois theory.* 50 minutes
BREAK 15:00UTC – Breakout rooms available.
15:30UTC Lindsay Childs, *The Galois Correspondence Ratio for some nilpotent $\mathbb{F}_p$-algebras.* 25 minutes
16:00UTC Andy Schultz, *How quaternion algebras shape the structure of square power classes over biquadratic extensions.* 25 minutes
BREAK 16:30UTC – Breakout rooms available.

Tuesday. Moderator: Tim Kohl

13:00UTC Andrea Caranti, *Finite p-groups, automorphisms, multiple holomorphs, and skew braces.* 50 minutes
14:00UTC Paul Truman, *Skew bracoids and Hopf-Galois structures on separable extensions.* 50 minutes
BREAK 15:00UTC – Breakout rooms available.
15:30UTC Alan Koch, *Skew left bracoids via abelian maps.* 25 minutes
16:00UTC Isabel Martin-Lyons, *Almost Classical Skew Bracoids.* 25 minutes
BREAK 16:30UTC – Breakout rooms available.

Wednesday. Moderator: Nigel Byott

13:00UTC Cindy Tsang, *Regular subgroups in the holomorph, fixed point free pairs of homomorphisms, and group factorizations.* 50 minutes
14:00UTC Ilaria Colazzo, *Derived-indecomposable solutions and skew braces with a finiteness condition.* 50 minutes.
BREAK 15:00UTC – Breakout rooms available.
15:30UTC Ilaria Del Corso, *Module braces: theory and applications.* 50 minutes.
BREAK 16:30UTC – Breakout rooms available.

Thursday. Moderator: Paul Truman

13:00UTC Kevin Keating, *Extensions of local fields in characteristic 0 and characteristic p.* 50 minutes
14:00UTC Nigel Byott, *Simple Skew Braces.* 50 minutes
BREAK 15:00UTC – Breakout rooms available.
15:30UTC Teresa Crespo, *Braces of size np.* 25 minutes.
16:00UTC Andrew Darlington, *The problem of finding almost-classically Galois extensions.* 25 minutes.
BREAK 16:30UTC – Breakout rooms available.

6:00PM CDT Dinner at Gorat's Steakhouse – 4917 Center St. Leave Scott Hall at 5:30pm. Walk 1.4 miles (31 min) east on Woolworth St, then Pine St. then south on 50th St, before turning east on Center.

Friday. Moderator: Kevin Keating

13:00UTC Daniel Gil-Muñoz, *A generalization of Kummer theory to Hopf-Galois extensions.* 50 minutes
14:00UTC Rob Underwood, *A Collection of Hopf orders in $K[C_{p^3}]$ in characteristic p.* 50 minutes
BREAK 15:00UTC – Breakout rooms available.
15:30UTC Griff Elder, *Through a dihedral prism.* 25 minutes.
BREAK 16:00UTC – Breakout rooms available.

Abstracts

Nigel Byott, University of Exeter.
Simple Skew Braces. 50 minutes
Abstract: Constructions for finite simple braces have been given by Bachiller (2018) and by Cedó, Jespers and Okniński (2020) via matched products. Little seems to be known, however, about simple skew braces which are not braces (i.e. which have non-abelian additive group), although there are examples where the multiplicative group is a finite simple group (and the additive group need not be simple). Recently Vendramin showed by a computer search that the smallest simple skew brace which is not a brace has order 12. In this talk, I will present a fairly explicit construction for a simple skew brace (which is not a brace) of size $p^p q$, where p, q are any primes such that q divides $p^p - 1$. Both the additive and multiplicative groups of these skew braces are soluble but not abelian. The smallest case $p = 2$, $q = 3$ gives a simple skew brace of size 12 as found by Vendramin. I will also mention some open problems connected with this construction.

Andrea Caranti, Università di Trento.
Finite p-groups, automorphisms, multiple holomorphs, and skew braces. 50 minutes
Abstract: The main aim of this expository talk is to share the knowledge of certain linear techniques that the author learned from Hermann Heineken in 1982/3. These techniques are useful in reducing the problem of computing the automorphism groups of certain finite p-group of nilpotence class two to a question in linear algebra. Recently, Cindy Tsang and the author have found a way to apply an extension of these techniques to the computation of the multiple holomorphs of certain p-group of nilpotence class two.

Lindsay Childs, University of Albany.
The Galois Correspondence Ratio for some nilpotent $\mathbb{F}_p$-algebras. 25 minutes
Abstract: A finite $\mathbb{F}_p$-algebra A with $A^e = 0$ yields a skew brace, hence a Hopf-Galois structure H of type $(A, +)$ on a G-Galois extension L/K of fields where $G \cong (A, \circ)$, the circle group of the radical algebra A. If $e \leq 3$, A defines a bi-skew brace. The Galois correspondence ratio (GCR) is the ratio (ideals of A)/(subgroups of $(A, \circ)$) that measures the proportion of subfields of L/K that are fixed under the action of sub-Hopf algebras of H. For commutative nilpotent $\mathbb{F}_p$-algebras of exponent $e < p$, Greither and the author [Publ. Mat. Debrecen, 2018] estimated the GCR and showed that it approaches 0 with increasing p. Applying some 2022 results of L. Stefanello and S. Trappeniers and some elementary techniques, we show that the two GCR's for the bi-skew brace defined by the non-commutative nilpotent $\mathbb{F}_p$-algebra A on n generators with $A^3 = 0$ are equal, and the GCR behaves like $4/p^{2n^3}$ for large n or p.

Ilaria Colazzo, University of Exeter.
Derived-indecomposable solutions and skew braces with a finiteness condition. 50 minutes.
Abstract: The problem of finding set-theoretic solutions to the Yang-Baxter equation goes back to Drinfeld. A first attempt to tackle this problem is studying indecomposable solutions.

This talk, based on joint work with M. Ferrara and M. Trombetti, will introduce skew braces in which every element has finitely many conjugates, which might be considered the brace-theoretical counterpart of FC-groups. Moreover, I will show that fundamental examples of such skew braces are structure groups of derived-indecomposable solutions (*i.e.* solutions whose derived is indecomposable).

Teresa Crespo, Universitat de Barcelona.
Braces of size np. 25 minutes.
Abstract: Let p be an odd prime number and let n be an integer not divisible by p and such that every group of order np has a normal subgroup of order p. (This holds in particular for $p > n$.) Under these hypotheses, we obtain a one-to-one correspondence between the isomorphism classes of braces of size np and the set of pairs $(B_n, [\tau])$, where $(B_n, +, \cdot)$ runs over the isomorphism classes of braces of size n and $[\tau]$ runs over the classes of group morphisms τ from $(B_n, \cdot)$ to $\mathbf{Z}_p^*$ under a certain equivalence relation. We have applied this result to classify left braces of sizes $8p$ (for $p \neq 3, 7$) and $12p$ (for $p \geq 7$). In my talk I will present this one-to-one correspondence and show its application to the construction of braces of size $12p$ in some examples.
Joint with Daniel Gil-Muñoz, Anna Rio, and Montserrat Vela

Andrew Darlington, University of Exeter.
The problem of finding almost-classically Galois extensions. 25 minutes.
Abstract: First talked about in Greither and Pareigis' fundamental 1987 paper, almost classically Galois extensions with group G and order n arise from the existence of regular subgroups of $\mathrm{Sym}(n)$ normalised by G and contained in G. In particular, they provide a large class of examples for which the so-called Hopf-Galois correspondence is bijective. In this talk, I will describe current work on obtaining all almost classically Galois extensions of a specified type N of order n, together with some results obtained so far. The hope being that the extra condition imposed will allow for such extensions to be found more efficiently than finding all transitive subgroups of the holomorph.

Ilaria Del Corso, Università di Pisa.
Module braces: theory and applications. 50 minutes.
Abstract: Last year, in this conference, I presented some result on the groups of units of rings, obtained by using methods that rely on braces.

After the conference I generalised those methods and better formalised the context, arriving to the definition of R-braces, which are braces whose additive group has also a module structure over a ring R, and for which the values of the gamma functions are automorphisms of R-modules. With this language a brace is a $\mathbb{Z}$-brace. The definition of R-brace generalises that of an F-brace, for F a field, already present in the literature.

In this talk we present a view on the theory of R-braces, showing that this class of braces enjoys all the natural properties one might expect. We then present two results which exhibit the advantages one can get when dealing with R-braces.

The first is the study of the splitting of a module brace in relation to the splitting of the ring R. This result is a generalisation of Byott's result on the splitting of a brace with nilpotent multiplicative group as a sum of its Sylow subgroups, and it is very natural and easy to obtain with our language.

The second result is a further generalisation of the main result of the paper by Featherstonhaugh, Caranti and Childs (TAMS 2012) which I refined from the one presented last year.

I will also give a converse of this result in the case of radical rings which allows us to make some progress on Fuchs' problem on the classification of the groups of units of rings.

Griff Elder, University of Nebraska at Omaha.
Through a dihedral prism. 25 minutes.
Abstract: Let D_8 be the symmetry group of the regular octagon. We consider the problem of determining the Galois module structure of the ring of integers in totally ramified D_8-extensions over a local fields of characteristic 2. This provides us with a nice, small setting for a discussion of: (a) the construction of extensions, (b) the resulting ramification sequence, (c) sufficient conditions for a Galois scaffold, and (d) applications of a scaffold. In many ways, the issues we face are generic – they are independent of Galois group and the prime characteristic. But not all.
Joint with Kevin Keating.

Daniel Gil-Muñoz, Charles University in Prague.
A generalization of Kummer theory to Hopf-Galois extensions. 50 minutes
Abstract: In this talk I will present a work in progress on a generalization of the notion of Kummer extension with the setting provided by Hopf-Galois theory. Classical Kummer extensions require that the ground field has characteristic coprime to n and contains the n-th roots of unity. We will see that such extensions can be characterized from the Galois action, and using this fact as an inspiration, I will define an H-Kummer condition for an H-Galois extension. This new notion does not require the ground field to contain roots of unity. Almost classically Galois extensions of this type correspond to radical extensions, i.e. those obtained from the adjunction of n-th roots of elements in the ground field. In order to move from pure extensions to radical ones, I will introduce a product of Hopf-Galois structures on almost classically Galois extensions under fairly general restrictions, which can be seen as an analogue of induced Hopf-Galois structures. Moreover, I will discuss some consequences on the module structure of H-Kummer extensions of number or p-adic fields.

Kevin Keating, University of Florida.
Extensions of local fields in characteristic 0 and characteristic p. 50 minutes
Abstract: There is a notion, dating back at least to Krasner, that a local field of characteristic 0 with large absolute ramification index approximates a field of characteristic p. Deligne made this idea precise in 1979 by giving a correspondence between certain extensions of local fields of characteristic 0 and local fields of characteristic p. In this expository talk I will outline Deligne's approach and give some examples to illustrate his method.
d'après P. Deligne

Alan Koch, Agnes Scott College.
Skew left bracoids via abelian maps. 50 minutes
Abstract: Skew left bracoids (formerly weak skew braces, neo skew braces) are a recent construction of Isabel Martin-Lyons and Paul Truman that generalize skew left braces, one which expands the relation between skew left braces and Hopf-Galois extensions on Galois extensions to potentially non-normal separable field extensions. We will show how one can use abelian maps (that is, endomorphisms of a group with abelian image) to construct a family of bracoids.

Tim Kohl, Boston University.
Beyond Isomorphic Holomorphs. 50 minutes
Abstract: For distinct groups G_1 and G_2 of the same order, one may have that their holomorphs $\{\mathrm{Hol}(G_i)\}$ are isomorphic. The classic examples of this are the dihedral and quaternionic groups, D_{2n} and Q_n, of order $4n$ for $n \geq 3$. The structural similarity of these groups partially explains why their holomorphs are isomorphic, but the 'counterexample' to this are the dihedral and quaternion groups D_4 and Q_2 of order 8, whose holomorphs don't even have the same size. Beyond these, there are groups, also of similar structure, for which their holomorphs are non-isomorphic (despite perhaps having the same size) but where their multiple holomorphs $\{\mathrm{NHol}(G_i)\}$ *are* isomorphic, which therefore exemplifies a different sort of equivalence. Going still further one may find examples of pairs (and more) of groups where neither the holomorphs, nor the multiple holomorphs are isomorphic, but rather whose quasiholomorphs, $\{\mathrm{QHol}(G_i)\}$, are isomorphic. And as the multiple holomorph and quasihololmorph parameterize certain classes of regular subgroups, this has implications in the enumeration of Hopf-Galois structures on Galois extensions, and in the enumeration/classification of skew-braces.

Isabel Martin-Lyons, Keele University.
Almost Classical Skew Bracoids. 25 minutes
Abstract: The skew bracoid (née weak skew brace) is a generalisation of the skew brace corresponding to Hopf-Galois structures on separable, but not necessarily normal field extensions. We investigate the situation in which such an extension L/K is almost classical, and pay special attention to the almost classical Hopf-Galois structures. Writing E for the Galois closure of L/K, these correspond under Greither-Pareigis theory to subgroups of the form $\lambda(H)^{opp}$ where H is a normal complement of $\mathrm{Gal}(E/L)$ in $\mathrm{Gal}(E/K)$. As on the Hopf-Galois side, this gives a particularly neat form of skew bracoid.

Andy Schultz, Wellesley College.
How quaternion algebras shape the structure of square power classes over biquadratic extensions. 25 minutes
Abstract: The Galois module structure for square power classes of a biquadratic extension have recently been determined. Although the modular representation theory allows for an infinite number of indecomposable types in this setting, it turns out that at most 9 summand types appear in this decomposition. The multiplicities of each summand type were originally determined according to the solvability of certain module-theoretic equations, but recently those conditions have been interpreted in terms of quaternion algebras. In this talk we survey these results and give some specific examples.

Lorenzo Stefanello, Università di Pisa.
Bi-skew braces in Hopf–Galois theory. 50 minutes
Abstract: The study of Hopf–Galois structures, valuable tools for dealing with classical problems in arithmetic or field theory in a more general context, has been enriched by an unexpected connection with skew braces, algebraic structures whose role in several areas of mathematics has been shown to be relevant.

The main goal of the talk is to explore the interpretation in Hopf–Galois theory of bi-skew braces, a class of skew braces recently studied with interest. We show that these objects can be effectively used to obtain Hopf–Galois structures for which the Hopf–Galois correspondence is surjective, and that the Hopf–Galois ratios of the two Hopf–Galois structures yielded by a bi-skew brace are nicely related. Moreover, we mention that when the Galois extension is an extension of p-adic fields, the Hopf–Galois structures given by bi-skew braces can be nicely 'split' in more manageable pieces.

A key ingredient to derive these result is a new version of the connection between Hopf–Galois structures and skew braces, obtained in a recent joint work with S. Trappeniers.
Joint with Senne Trappeniers.

Paul Truman, Keele University.
Skew bracoids and Hopf-Galois structures on separable extensions. 50 minutes
Abstract: Skew bracoids (née weak skew braces) are a generalization of skew braces that correspond to Hopf-Galois structures on separable, but potentially non-normal, field extensions. We will give an updated and refined interpretation of this correspondence, following the approach of Stefanello and Trappeniers in the Galois case. In particular, we will study generalizations of some of their results concerning the Hopf-Galois correspondence, and show that various properties of intermediate fields (such as being Galois or Hopf-Galois extensions of the base field) are reflected in various notions of ideal in a skew bracoid.
Joint with Isabel Martin-Lyons

Cindy Tsang, Ochanomizu University (Japan).
Regular subgroups in the holomorph, fixed point free pairs of homomorphisms, and group factorizations. 50 minutes
Abstract: By previous work of Byott and Childs, fixed point free pairs of homomorphisms and group factorizations may be used to construct regular subgroups contained in the holomorph $\mathrm{Hol}(N)$ of a finite group N. In this talk, we shall first review this construction and then explain how it can be generalized to construct all regular subgroups of $\mathrm{Hol}(N)$ when N has trivial center. We shall also discuss how one can apply this to give a complete characterization of the finite non-abelian simple groups N for which $\mathrm{Hol}(N)$ contains a solvable regular subgroup.

Rob Underwood, Auburn University at Montgomery.
A Collection of Hopf orders in $K[C_{p^3}]$ in characteristic p. 50 minutes
Abstract: Let p be a prime number, let K be a field of characteristic p that is complete with respect to a discrete valuation and let C_p^3 denote the elementary abelian group of order p^3. A recent paper (2022) has classified all of the Hopf orders in the K-Hopf algebra $K[C_p^3]$. We use this result to construct a collection of Hopf orders in $K[C_{p^3}]$ where C_{p^3} is cyclic of order p^3.