

THE INTERPLAY BETWEEN SKEW BRACES AND HOPF-GALOIS THEORY

KEELE UNIVERSITY (COLIN REEVES BUILDING, ROOM CR114)
TUESDAY 18TH APRIL 2023

SCHEDULE (TIMINGS ARE BST (GMT +1))

- 9:00am** Vendramin: *Some problems on skew braces*
9:30am Van Antwerpen: *Nilpotency in skew braces: an overview*
10:30am Coffee
11:00am Properzi: *A graph associated with finite skew braces*
12:00pm Colazzo: *The structure monoid of set-theoretic solutions to the YBE*
1:00pm Lunch
2:00pm Stefanello: *Skew braces and the Hopf-Galois correspondence*
3:00pm Tea
3:30pm Truman: *Skew bracoids and Hopf-Galois structures on separable extensions*

ABSTRACTS

Arne van Antwerpen, Vrije Universiteit Brussel

Nilpotency in skew braces: an overview

50 minutes

Arne.van.Antwerpen@vub.be

Skew braces are an incredibly effective tool to study bijective non-degenerate solutions of the Yang-Baxter equation. In this talk we examine the known notions of nilpotency of skew braces and link some of these to properties of the associated solutions. First, we deal with left nilpotent skew braces of nilpotent type, which corresponds to skew braces with multiplicative nilpotent group. In particular, these skew braces can be decomposed as the direct product of skew braces of prime power order. Secondly, we connect right nilpotent skew braces with multipermutation solutions. Last, we present centrally nilpotent skew braces, which behave similarly to nilpotent groups and are a large class of skew braces that could be attacked using techniques from nilpotent groups.

Ilaria Colazzo, University of Exeter*The structure monoid of set-theoretic solutions to the YBE*

50 minutes

I.Colazzo@exeter.ac.uk

In this talk, we will introduce the structure monoid of a set-theoretic solution to the Yang-Baxter equation and explore some combinatorial aspects. We will show that the structure monoid is useful for investigating bijective non-degenerate solutions to the Yang-Baxter equation. This talk is based on joint work with Jespers, Van Antwerpen and Verwimp and an ongoing project with Kubat, Jespers, and Van Antwerpen.

Silvia Properzi, Vrije Universiteit Brussel*A graph associated with finite skew braces*

50 minutes

Silvia.Properzi@vub.be

Coauthors: Arne van Antwerpen.

Inspired by a work of Bertram, Herzog, and Mann, we introduce a graph associated with the λ -classes of a given finite skew brace. Given a finite skew brace A , its *graph of non-trivial λ -classes* has as vertices the non-trivial λ -orbits of A and two vertices are adjacent if their sizes are not coprime. This graph has similar properties to the one defined for groups by Bertram, Herzog, and Mann: it has at most two connected components, some bounds on the diameter and there is only one skew brace whose graph consists of two disconnected vertices. On the other hand, in contrast to the group version, there exist graphs with precisely one vertex, and skew braces with this property can be classified completely.

Lorenzo Stefanello, Università di Pisa*Skew braces and the Hopf-Galois correspondence*

50 minutes

Lorenzo.Stefanello@phd.unipi.it

Coauthors: Senne Trappeniers

Hopf-Galois theory, a generalisation of Galois theory in which the action of a Galois group is replaced by a suitable action of a Hopf algebra, has been showed to be valuable in dealing with classical arithmetic problems in a more general context. An open problem in Hopf-Galois theory regards the surjectivity of the Hopf-Galois correspondence; the well-know bijective Galois correspondence between subgroups of the Galois group and intermediate fields can be generalised to Hopf-Galois structures, but the correspondence one obtains is injective but not necessarily surjective. Very few examples

where this correspondence is surjective are known; it is of interest to find new ones, and to understand more in general why it should be (or not be) surjective. The goal of this talk is to approach this problem using the point of view of skew braces. We show that using a new version of connection between skew braces and Hopf-Galois structures, recently developed with S. Trappeniers, we can translate the problem of the surjectivity of the Hopf-Galois correspondence in a natural problem in skew brace theory, thereby deriving several new examples of Hopf-Galois structures for which the Hopf-Galois correspondence is surjective.

Paul Truman, Keele University

Skew bracoids and Hopf-Galois structures on separable extensions

50 minutes

`P.J.Truman@Keele.ac.uk`

Coauthors: Isabel Martin-Lyons

Skew bracoids (née weak skew braces) are a generalization of skew braces that correspond to Hopf-Galois structures on separable, but potentially non-normal, field extensions. We will explain this correspondence, following the approach of Stefanello and Trappeniers in the Galois case. In particular, we will study generalizations of some of their results concerning the Hopf-Galois correspondence, and show that various properties of intermediate fields (such as being Galois or Hopf-Galois extensions of the base field) are reflected in various notions of ideal in a skew bracoid.

Leandro Vendramin, Vrije Universiteit Brussel

Some problems on skew braces

25 minutes

`Leandro.Vendramin@vub.be`

In this talk, I will discuss some problems in the theory of skew braces and possible applications to other research areas.