SKEW BRACES, SKEW BRACOIDS, AND HOPF-GALOIS THEORY

KEELE UNIVERSITY
DENISE COATES BUILDING, ROOM LG008
WEDNESDAY 2$^{\text{ND}}$ - FRIDAY 4$^{\text{TH}}$ AUGUST, 2023

SCHEDULE

Wednesday 2$^{\text{nd}}$ August, afternoon. Chair: Isabel Martin-Lyons

14:00 Welcome from Prof. Daniel Reidenbach, Head of School of Computer Science and Mathematics.

14:10 Ilaria Del Corso: *Hopf-Galois structures on extensions of degree p^2q and skew braces of order p^2q*

15:00 Anna Rio: *Schur-Zassenhaus on skew braces*

15:50 Tea

16:30 Lorenzo Stefanello: *On notions of solubility in Hopf-Galois theory*

Thursday 3$^{\text{rd}}$ August, morning. Chair: Andrew Darlington

09:00 Paul Truman: *Canonical ideals in skew bracoids*

09:50 Ilaria Colazzo: *Skew bracoids and solutions to the Yang-Baxter equation*

10:20 Coffee

11:00 Isabel Martin-Lyons: *Homomorphisms and short exact sequences of skew bracoids*

Thursday 3$^{\text{rd}}$ August, afternoon. Chair: Daniel Gil-Muñoz

14:00 Alan Koch: *Skew bracoid webs arising from abelian maps*

14:50 Andrew Darlington: *How to construct a database of Hopf-Galois structures of small degree*

15:20 Tea

Friday 4$^{\text{th}}$ August, morning. Chair: Lorenzo Stefanello

09:00 Tomasz Brzeziński: *Lie trusses*

09:50 Nafeesa Khalil: *Twisting biquadratic extensions*

10:20 Coffee

11:00 Nigel Byott: *An approach to a conjecture of Rump on quasi-linear cycle sets of prime cardinality*

ABSTRACTS

Tomasz Brzeziński, Swansea University & University of Białystok
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Lie trusses
40 minutes
Coauthor: James Papworth

In 2003 Grabowska, Grabowski and Urbański proposed the definition of a Lie bracket on an affine space. This definition relies on the use of the vector space underlying an affine space. However, an affine space can be defined with no reference to a vector space provided one uses heaps rather than groups. In this talk we make a proposal of an intrinsically affine definition of a Lie bracket. More precisely, we define a Lie bracket on an abelian heap (also with an action of a commutative ring, thus in particular on an affine space). This definition is compared to that of Grabowska, Grabowski and Urbański and illustrated by a number of examples.

Nigel Byott, University of Exeter
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An approach to a conjecture of Rump on quasi-linear cycle sets of prime cardinality
40 minutes

A conjecture of Rump states that the set-theoretic solutions of the Yang-Baxter equation arising from finite quasi-linear cycle sets are multipermutational. For solutions of prime cardinality p, this amounts to the following statement:

CONJECTURE 1 (Rump): *Let τ be a permutation of the group $A = C_p$ with $\tau(0) = 0$, and define a new binary operation on A by*

$$a \cdot b = \tau(b - a) - \tau(-a).$$

If

$$\tau(a) \cdot \tau(b) = \tau(a \cdot b) \; \forall a, b, \in A,$$

then τ is an automorphism of the group A.

In this talk, I discuss the group

$$\mathrm{Aut}(A, \cdot) = \{\sigma \in \mathrm{Sym}(A) : \sigma(0) = 0, \; \sigma(a) \cdot \sigma(b) = \sigma(a \cdot b) \; \forall a, b, \in A\}$$

of automorphisms of the pointed magma $(A, \cdot)$, and propose the following refinement of Conjecture 1:

CONJECTURE 2: *If $\sigma \in \mathrm{Aut}(A, \cdot)$ then either σ or τ is an automorphism of the group A.*

Ilaria Colazzo, University of Exeter

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Skew bracoids and solutions to the Yang-Baxter equation

20 minutes

Coauthors: Isabel Martin-Lyons and Paul Truman

Is it possible to associate a skew bracoid with a solution to the Yang-Baxter equation? In this talk, we will answer the question for skew bracoids which are quotients of skew braces by strong left ideals.

Andrew Darlington, University of Exeter

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How to construct a database of Hopf-Galois structures of small degree.

20 minutes

Computational methods have been used to compute both Hopf-Galois structures on low degree Galois extensions and skew braces of small order. Subsequent work by Crespo and Salguero has given results on low degree separable (but not necessarily normal) extensions (up to degree 31). This talk will introduce and summarise current efforts to use a new approach to obtain further results in the separable case. I will finish by explaining what still needs to be done as well as giving a few possible applications.

Ilaria Del Corso, Università di Pisa

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Hopf-Galois structures on extensions of degree p^2q and skew braces of order p^2q

40 minutes

Coauthors: Elena Campedel and Andrea Caranti

In a joint project with Elena Campedel and Andrea Caranti, we classified all Hopf Galois structure on extensions of degree p^2q and the skew braces of order p^2q, for any pair of distinct primes p, q with $p > 2$. Our method relies on the computation of all the functions $\gamma : G \to \mathrm{Aut}(G)$ satisfying $\gamma_{x \cdot \gamma_x(y)} = \gamma_x \gamma_y$ for all $x, y \in G$, for all groups G of order p^2q. These maps are related to the regular subgroups of the holomorph of G (and hence to the Hopf Galois structures). In this talk I will present our results, our approach and some tools we developed to get the enumeration.

4

Nafeesa Khalil, University of Manchester
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Twisting biquadratic extensions
20 minutes

Given a Hopf algebra H, a 2-cocycle μ of H and a H-module algebra A, Drinfeld and Majid introduced a notion of twist that can be used to construct a Hopf algebra H_μ and a H_μ-module algebra A_μ. For a quasitriangular Hopf algebra H, the quasitriangular structure gives such a twist. The structures arising from this process are not necessarily (co)commutative and can lead to Hopf-Galois structures of noncommutative algebras as per the definition of Kreimer and Takeuchi. This has potential to assist in translating ideas from algebraic number theory into a noncommutative setting. In this talk I will present a twist of a biquadratic extension that results in Hopf-Galois structures on quaternion algebras over a field.

Alan Koch, Agnes Scott College
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Skew bracoid webs arising from abelian maps
40 minutes
Coauthor: Paul J.Truman

We introduce the notion of bracoid webs, an analogue of brace blocks. For G any group and $\psi : G \to G$ an endomorphism with abelian image we construct collections of groups $\{G_i : i \in \mathbb{Z}^+\}$, $\{N_j : j \in \mathbb{Z}^+\}$ such that (G_i, N_j) is a bracoid for all i, j. Each bracoid web gives rise to numerous Hopf-Galois structures on various separable field extensions; we describe these constructions with explicit Greither-Pareigis computations. Many examples will be given.

Isabel Martin-Lyons, Keele University
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Homomorphisms and short exact sequences of skew bracoids
40 minutes

The skew bracoid is a generalisation of the skew brace corresponding to Hopf-Galois structures on separable, but not necessarily normal field extensions. We define homomorphisms and isomorphisms of these objects, taking care to distinguish isomorphism from our notion of equivalence. Expanding on these ideas, we outline what a short exact sequence of skew bracoids should look like with a focus on the case in which the term on the left is almost classical and the central term is induced.

Anna Rio, Universitat Politcnica de Catalunya

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Schur-Zassenhaus on skew braces

40 minutes

Coauthors: Teresa Crespo, Daniel Gil-Muñoz, Montse Vela

We consider braces of size np, where p is a prime and n is an integer not divisible by p and such that its only divisor congruent to 1 mod p is 1. We define the double semidirect product of skew braces and prove that any skew brace of size np is a double semidirect product of the trivial brace of size p and a skew brace of size n. We develop an algorithm to obtain all skew braces of size np from the set of skew braces of size n and provide a formula to count them. We use the result to describe all skew braces of size $12p$ for $p \geq 7$ and prove a conjecture of Bardakov, M.V. Neshchadim and M.K. Yadav.

Lorenzo Stefanello, Università di Pisa

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On notions of solubility in Hopf-Galois theory

40 minutes

Coauthors: Ilaria Del Corso and Paul J. Truman

HopfGalois structures on Galois extensions are described by the interactions of a field extension, a (Galois) group, a Hopf algebra, and a skew brace. In this talk we discuss connections between notions of solubility for these objects. As a consequence, we show that we can control and describe certain HopfGalois structures in more manageable ways.

Paul Truman, Keele University

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Canonical ideals in skew bracoids

40 minutes

Coauthor: Isabel Martin-Lyons

We review the definition of a skew bracoid and some fundamental notions including equivalence, γ-functions, and ideals. We then show that various canonical constructions of ideals in a skew brace, such as the fix and the socle, have natural skew bracoid generalizations. Finally, we begin to explore whether certain chains of ideals might allow us to formulate notions of nilpotency or solubility for skew bracoids.