SCHEDULE: HOPF ALGEBRAS AND GALOIS MODULE THEORY, MAY 22–26, 2017

Monday.

9:00am *Welcome*, Dave Boocker (Dean, College of Arts & Sciences)

9:30am Koch: *A characteristic-independent description of Hopf orders using Breuil-Kisin modules.* 60 min.

11:00am Elder: *Through a looking glass: the Heisenberg group modulo an odd prime p.* 60 minutes

1:30pm Underwood: *Singular weak quasitriangular structures.* 30 minutes

2:30pm Childs: *Introduction: Nilpotent algebras and Hopf Galois extensions.* 60 minutes

Tuesday.

9:30am Childs: *Quantifying the failure of surjectivity of the Galois correspondence for Hopf Galois extensions.* 60 minutes

11:00am Truman: *Hopf-Galois module structure: Tidying up some tame extensions.* 60 minutes

1:30pm Taylor: *Hopf-Galois structures on quaternionic extensions.* 30 minutes

Evening One of only 4 remaining great Omaha Steakhouses, Johnny's Cafe `http://www.johnnyscafe.com/`, as featured in the 2002 Alexander Payne movie, *About Schmidt.*

Wednesday.

9:30am Kohl: *Normality and short exact sequences in Hopf-Galois extensions.* 60 minutes

11:00am Underwood: *The structure of Greither-Pareigis Hopf algebras.* 60 minutes

1:30pm Byott: *Braces and Hopf-Galois structures.* 60 minutes

Evening Informal dinner: potroast, veg chili (both from slow cooker), mash potatoes and green beans.

Thursday.

9:30am Keating: *Perturbing Eisenstein polynomials over local fields* 60 minutes

11:00am Kohl: *A class of profinite Hopf-Galois extensions over* $\mathbb{Q}$. 60 minutes

Friday.

9:30am Byott: *Lambda-divided power Hopf algebras.* 60 minutes

Afternoon Group bike ride on the Keystone Trail.

Evening Pool party at the Elder-berry Residence: 5624 Leavenworth St.

Nigel Byott, University of Exeter.

Braces and Hopf-Galois structures. 60 minutes

Abstract: A *brace* is a set B with operations $+$ and $\cdot$ so that $(B,+)$ is an abelian group, $(B,\cdot)$ is a group, and $a\cdot(b+c)+a = a\cdot b+a\cdot c$ for all a, b, $c \in B$. Braces were introduced by Wolfgang Rump in 2007 to study certain solutions of the quantum Yang-Baxter equation, and have since been investigated by a number of authors. It turns out that a finite group G arises as the multiplicative group $(B,\cdot)$ of some brace B if and only if a Galois extension of fields with Galois group G admits a Hopf-Galois structure of abelian type. I will describe how braces are related to the Yang-Baxter equation, how they are related to Hopf-Galois structures, and how some results in the literature on braces translate to apparently new results on Hopf-Galois structures. One example of this is that, for any sufficiently large prime p, there is a (nonabelian) group G of order p^{10} and exponent p such that a Galois extension with group G does not admit any Hopf-Galois structures of abelian type.

Lambda-divided power Hopf algebras. 60 minutes

Abstract: Motivated by previous work on Baxter algebras, Andrews, Guo, Keigher and Ono (2005) defined the λ-divided power Hopf algebra $\mathcal{A}_\lambda$ over an arbitrary commutative base ring R. (Taking the parameter λ to be 0 gives the usual divided power Hopf algebra.) Their proof that $\mathcal{A}_\lambda$ is indeed a Hopf algebra involves lengthy direct verification of the axioms using nontrivial combinatorial identities. I will show how $\mathcal{A}_\lambda$ arises more naturally as the dual of a one-dimensional polynomial formal group. Taking R to be a local field of characteristic $p > 0$, and applying standard constructions for formal groups, we can obtain field extensions of degree p^m, for arbitrary $m \geq 1$, which are Hopf-Galois extensions over a truncation $\mathcal{A}_{\lambda,m}$ of $\mathcal{A}_\lambda$. These extensions admit scaffolds in $\mathcal{A}_{\lambda,m}$, allowing their integral "Galois" module structure to be studied. This is joint work with Griff Elder and Alan Koch.

Lindsay Childs, University of Albany.

Quantifying the failure of surjectivity of the Galois correspondence for Hopf Galois extensions. 60 minutes

Abstract: Let A be a finite commutative nilpotent F_p-algebra structure on G, an elementary abelian group of order p^n. If K/k is a Galois extension of fields with Galois group G and $A^p = 0$, then corresponding to A is an H-Hopf Galois structure on K/k of type G. For that Hopf Galois structure we may study the image of the Galois correspondence from k-subHopf algebras of H to intermediate subfields of K containing k by utilizing the fact that the intermediate subfields correspond to the F_p-subspaces of A, while the subHopf algebras of

H correspond to the ideals of A. We obtain upper and lower bounds on the proportion of subspaces of A that are ideals of A, and test the bounds on some examples. This is joint work with C. Greither.
Introduction: Nilpotent algebras and Hopf Galois extensions. 60 minutes
Abstract:

Griff Elder, University of Nebraska at Omaha.

Through a looking glass: the Heisenberg group modulo an odd prime p. 60 minutes

Abstract: In this talk, I will restrict myself to the nonabelian group $H(p)$ of order p^3 and exponent p, and for this group, take a stroll through all the various topics we consider each May. This is work in progress. The number of Hopf-Galois structures is known due to work in progress by Kayvan Zenouz (University of Exeter). Restricting now to totally ramified Galois extensions L with Galois group $H(p)$ over a local field K of characteristic p, I will determine all such extensions (there is no obstruction to the embedding problem), and explicitly describe the ramification breaks for each extension. This is unexpectedly complicated. I will then introduce a subclass of such extensions that have a Galois scaffold. For these extensions, Galois module structure is known, and this suggests a form that the Hopf orders in $KH(p)$ may take. I will then verify that these do indeed provide a family of Hopf orders in $KH(p)$.

Kevin Keating, University of Florida.

Perturbing Eisenstein polynomials over local fields 60 minutes

Abstract: Let K be a local field whose residue field has characteristic p and let L/K be a finite separable totally ramified extension. Let π_L be a uniformizer for L and let $f(X)$ be the minimum polynomial for π_L over K. Suppose $\tilde{\pi}_L$ is another uniformizer for L such that $\tilde{\pi}_L \equiv \pi_L + r\pi_L^{\ell+1} \pmod{\pi_L^{\ell+2}}$ for some $\ell \geq 1$ and $r \in \mathfrak{O}_K$. Let $\tilde{f}(X)$ be the minimum polynomial for $\tilde{\pi}_L$ over K. In this talk I will give congruences for the coefficients of $\tilde{f}(X)$ in terms of ℓ, r, and the coefficients of $f(X)$. These congruences improve and extend work of Krasner.

Alan Koch, Agnes Scott College.

A characteristic-independent description of Hopf orders using Breuil-Kisin modules. 60 minutes

Abstract: Let K be a non-Archimedean local field with valuation ring $\mathfrak{O}_K$ and let p be the residue field characteristic. Let H be a finite, flat, commutative, cocommutative K-Hopf algebra of p-power rank. In an effort to construct a unified theory, we will show how Breuil-Kisin modules can be used to classify $\mathfrak{O}_K$-Hopf

orders in H regardless of the characteristic of K. Special cases of our construction provide alternate descriptions of Hopf orders in many well-known situations, including (a) H primitively generated and $\mathrm{char}(K) = p$; (b) $H = KC_p^n$, $\mathrm{char}\, K = 0$; and (c) $H = KC_{p^n}$, $\mathrm{char}\, K = 0$. New examples will also be given.

Tim Kohl, Boston University.

Normality and short exact sequences in Hopf-Galois extensions. 60 minutes

Abstract: Let K/k be a Galois extension where $G = Gal(K/k)$ which is also Hopf-Galois with respect to the action of $H_N = (K[N])^{\lambda(G)}$ for $N \leq Perm(G)$ a regular subgroup normalized by $\lambda(G)$. If $P \lhd N$ is also normalized by $\lambda(G)$ then $H_P = (K[P])^{\lambda(G)}$ is a k-Hopf algebra and by Chase and Sweedler K/F is Hopf-Galois with respect to $F \otimes H_P$ where F is K^{H_P}. What we are interested in is the relationship between $G' = Gal(K/F)$ and P since both act on K/F and moreover whether N/P acts on F/k to make it Hopf-Galois. Concordant with this is examining the parallel between the exact sequence $1 \to k[G'] \to k[G] \to k[G/G'] \to 1$ of Hopf algebras (which act by means of the classical actions on K and F) and the corresponding sequence $1 \to (K[P])^{\lambda(G)} \to (K[N])^{\lambda(G)} \to (K[N/P])^{\lambda(G)/\lambda(G')} \to 1$ arising from the actions due to these (semi)-regular subgroups.

A class of profinite Hopf-Galois extensions over $\mathbb{Q}$. 60 minutes

Abstract: For p a prime and $a \in \mathbb{Q}$, where a is not a p^n-th power of any rational number, the extension $\mathbb{Q}(w_n)/\mathbb{Q}$ where $w_n = \sqrt[p^n]{a}$ is separable but non-normal. The Hopf-Galois theory for separable extensions was determined by Greither and Pareigis, and the specific classification for radical extensions such as these by the author. In this work we extend this theory to a certain class of profinite extensions, namely those formed from the union of these $\mathbb{Q}(w_n)$. We construct a 'profinite' Hopf algebra which acts, and show that it satisfies a generalization of a result due to Haggenmüller and Pareigis on the structure of Hopf algebra forms of group algebras.

Stu Taylor, Keele University.

Hopf-Galois structures on quaternionic extensions. 30 minutes

Abstract: Quaternionic extensions of number fields (with Galois group isomorphic to the quaternion group of order 8) have been important in the history of Galois module structure. For example, there exist tamely ramified quaternionic extensions $L/\mathbb{Q}$ which have local, but not global, normal integral bases. We might wonder whether the algebraic integers in such extensions are more satisfactorily described via a non-classical Hopf-Galois structure acting on the extension. In this talk, we determine all of the Hopf-Galois structures

admitted by a Galois extension of fields $L/\mathbb{Q}$ with Galois group isomorphic to the quaternion group of order 8, and study the $\mathbb{Q}$-algebra structure of the Hopf algebras involved.

Paul Truman, Keele University.

Hopf-Galois module structure: Tidying up some tame extensions. 60 minutes

Abstract: Over the past few years we have developed some tools for analysing the Hopf-Galois module structure of the ring of algebraic integers of a tamely ramified extension of local fields, and presented evidence for a conjecture that it is a free module over its associated order in all of the Hopf-Galois structures admitted by the extension. In this talk we let p, q, r be prime numbers and study tamely ramified Galois extensions L/K of a p-adic fields of degree qr. We show how the tools we have developed can be applied to make the study of the Hopf-Galois module structure of $\mathfrak{O}_L$ (almost) painless.

Rob Underwood, Auburn University at Montgomery.

The structure of Greither-Pareigis Hopf algebras. 60 minutes

Abstract: Let $L/\mathbb{Q}$ be a Galois extension with group G where G has order n, $2 \leq n \leq 6$. Let N be a regular subgroup of $\mathrm{Perm}(G)$ normalized by $\lambda(G)$, and let $H = (LN)^G$ be the Greither-Pareigis Hopf algebra acting on $L/\mathbb{Q}$. In this talk we review various results on the structure of H, including a new result on the structure of $H = (L\lambda(S_3))^{S_3}$.

Singular weak quasitriangular structures. 30 minutes

Abstract: Let K be a field, let $n \geq 1$, and let $M = \{1, a, a^2, \ldots, a^n\}$ be the monoid with multiplication defined as $a^i a^j = a^{i+j}$ if $i + j \leq n$ and $a^i a^j = a^n$ if $i + j > n$. Let KM be the monoid bialgebra with linear dual KM^D. For $n = 1, 2$, we construct all of the weak quasitriangular pairs (KM^D, R) for which R is singular, i.e., R is not a unit in $KM^D \otimes KM^D$. These singular weak quasitriangular structures correspond to non-trivial bimorphisms $M \times M \to K$, which may be of some interest when considering the structure of KM.

1. Additional Information

Design of conference. Talks are either 30 or 60 minutes.

In previous conferences, one of the features that everyone has enjoyed has been the inclusion of guiding principles, half-backed ideas, crazy conjectures. Please come with some to share.

Lecture space. The talks will be in Durham Science Center Room 254. There is a computer and projector connected to the Internet, along with a regular blackboard. So, if you would like, you can use both at the same time. There is also an ELMO, a document camera for projecting images from paper. And if you would like to use transparencies, it is possible with an advance request.

Coffee, snacks & lunch. There is a Starbucks in the Library next door to Durham Science Center that is open 10:30-2:00 each day. But we will also have a coffee machine in the room, pitcher of water/glasses along with fruit (apples, oranges, bananas) and bagels with cream cheese.

Lunch. The Food court in the Milo Bail Student center will be open from 7:30 till 2:00pm each day (Mexican, Asian and Italian food, as well as burgers, subway sandwiches, etc). Other options include Fuddruckers, Noodles & Company, Vietnamese-Thai Restaurant all slightly west of campus on 72nd St.

Wi-Fi. There is Wireless available, including *eduroam*. Visit `http://www.unomaha.edu/information-services/networks-and-connectivity.php`